RISE PARK PRIMARY AND NURSERY SCHOOL

Calculation Policy

September 2022

Signed by Chair of Committee	
Print Name	Jeanette Kirkby
Date	September 2022
Date of review	September 2024

Overview of our policy

Our policy follows guidelines set out by the national curriculum for mathematics, whose aim is to ensure that all pupils:

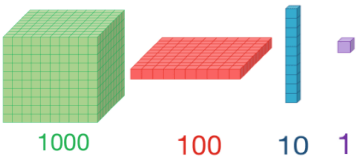
* Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
* **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.
*Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Our calculation policy builds on the children's solid understanding of place value, connections between numbers and the 4 operations, number fluency and number sense. Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

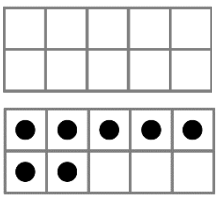
Children need to progress by using concrete resources, then relating these to pictorial representations and finally abstract representations, such as a typical number calculation (34 + 7). This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (Numicon, Dienes (Base ten), HTO counters, cubes, etc.) helps reinforce understanding and provide support when calculating mentally, with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part-part whole diagrams are also recommended.

Resources and visual used:

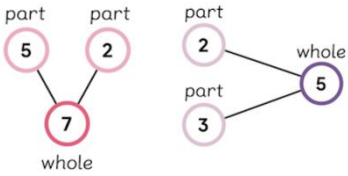
Dienes (Base 10)



Ten Frames

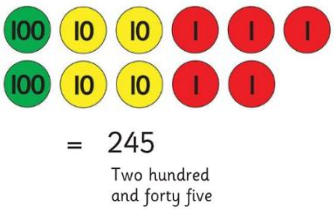


Part-part-whole model

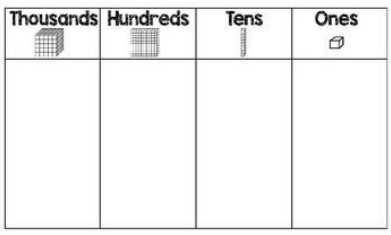


Place value arrows

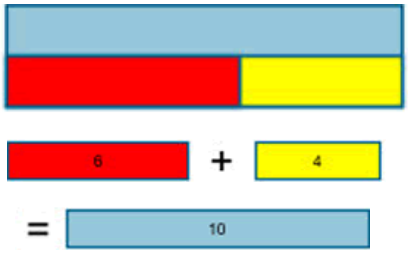
Place value counters



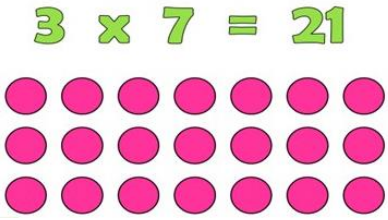
Place value grid



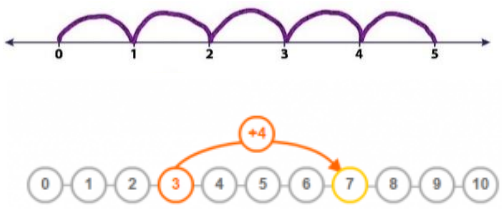
Bar models



Arrays



Number lines



Year One

Addition:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count up with confidence

Continuing on from Early Years, children should feel confident counting forwards and backwards through the number system, up to and beyond 100. This needs to be embedded before children can learn to add effectively.

Combine and count

Children should be able to combine two sets of objects (aggregation) which will then progress onto adding on to a set (augmentation)

They should also understand the commutative law that numbers can be re-ordered and the total is the same.

(5 + 7 = 12)

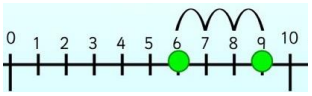
(7 + 5 = 12)

Use a range of different resources to support

Children should have access to a wide range of counting equipment, everyday objects, number tracks, number lines, and be shown numbers in different contexts.

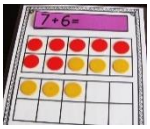
Number lines and number tracks

Children should have access to number lines and number tracks. These can be used to show the order of numbers and then be used to help children count on. Children should be able to start at the larger number and count on from there.



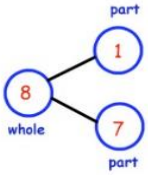
Ten frames

Use 10 frames to add within and beyond 10.
Use different coloured counters if possible.



Part-part-whole models

Children should be able to use the vocabulary terms 'part' and 'whole' when adding together numbers. The part-part-whole model should be used with concrete objects, pictures and then with digits.



Calculation symbols

Children should be able to recognise, understand, read and write the + and = symbols. They should be able to use these to form their own calculations.

Missing number calculations

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 2 = 1 + 1 2 + 3 = 4 + 1

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 3 + 4 = ___ ___= 3 + 4 3 + ___= 7 7 = ___+ 4

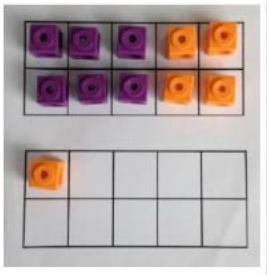
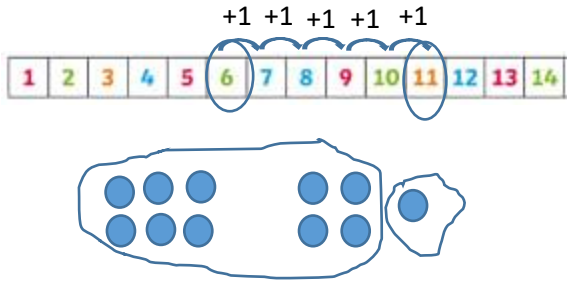
Explore with numbers

Children should be given the chance to explore with adding up numbers. E.g. Give children 10 blocks – what addition calculations can they make? Can they notice all answers must make 10?

1 + 9 = 10 9 + 1 = 10 2 + 8 = 10 8 + 2 = 10 and so on.

Concrete → Pictorial → Abstract example:

6 + 5 = 11

Concrete:	Pictorial:	Abstract:
		5 + 6 = 11 6 + 5 = 11 6 and 5 more makes 11

Key vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation

Key skills:

- Read and write numbers to 100 in numerals, including 1–20 in words.
- Recall bonds to 10 and 20, and addition facts within 20.
- Count to and across 100.
- Count in multiples of 1, 2, 5 and 10 to 100.
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Subtraction:

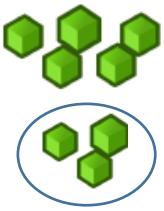
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count back with confidence

Continuing on from Early Years, children should feel confident counting forwards and backwards through the number system, up to and beyond 100. This needs to be embedded before children can learn to add effectively.

Move and count

Children should be taught to move the objects away one by one when subtracting. They must understand that the number of objects left that have not been moved shows us the answer.



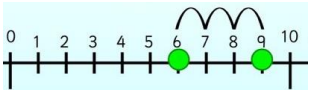
5 – 2 = 3

Use a range of different resources to support

Children should have access to a wide range of counting equipment, everyday objects, number tracks, number lines, and be shown numbers in different contexts.

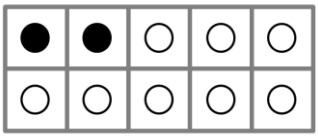
Number lines and number tracks

Children should have access to number lines and number tracks. These can be used to show the order of numbers and then be used to help children count back. Children should be able to start with the total and count back from there.



Ten frames

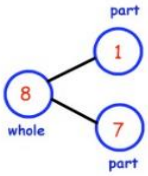
Use 10 frames to subtract from 20 or less.
Use different coloured counters if possible.



(10 – 8 = 2)

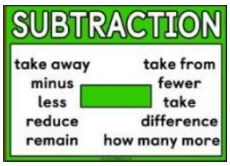
Part-part-whole models

Children should be able to use the vocabulary terms 'part' and 'whole' when adding together numbers. The part-part-whole model should be used with concrete objects, pictures and then with digits.



Calculation symbols

Children should be able to recognise, understand, read and write the - and = symbols. They should be able to use these to form their own calculations.



Missing number calculations

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 9 = 12 - 3 6 – 2 = 5 - 1

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 9 - 8 = [] [] = 9 - 8 8 + [] = 9 9 = [] + 8

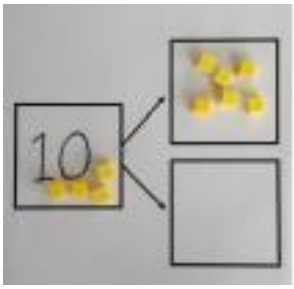
Explore with numbers

Children should be given the chance to explore with subtracting numbers.

E.g. Start with 20 blocks. What happens when we take 1 away each time? 2 away each time? 5 away each time? What patterns can you notice? Write down the calculations – can you see the pattern now?

Concrete → Pictorial → Abstract example:

10 – 6 = 4

Concrete:	Pictorial:	Abstract:
		10 – 6 = 4 10 - 4 = 6 10 subtract 6 makes 4 6 less than 10 is 4

Key vocabulary:

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation

Key skills:

- Given a number, say one more or one less.
- Count to and over 100, forward and back, from any number.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

Multiplication:

Counting in equal groups

Children should understand that multiplication requires counting in equal groups. Children should be encouraged to make equal groups with a range of resources e.g. Can you show me how to make cubes in towers of 2, put the teddy bears in groups of 3?

Combine groups of the same size (repeated addition)

Multiplication builds on children's knowledge of addition, in that it is repeated addition. Once children have made the equal groups, ask how many groups and then how many total.

E.g. How many towers of 2 cubes do you have? How many cubes are there altogether? How can we write this?

2 + 2 + 2 + 2 = 8

2 x 4 = 8 (2 four times = 8)

Relate to doubling

Allow children to make the connection between doubling and multiplying by 2.

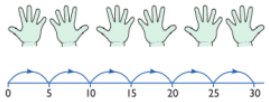
E.g. What is double 3? 6. That's 2 lots of 3s. That looks like this.

How can we say that another way?

Double 3 = 6 → 3 x 2 = 6

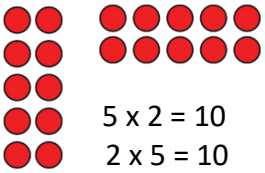
Focus on 2s, 5s and 10s

Many of the examples given should be in groups of 2s, 5s and 10s, as highlighted in the curriculum. This is because children need to know these multiplication facts by the end of year 2. Children should be frequently counting in steps of 2, 5 and 10 in their maths starters in lessons.



Arrays

Arrays should be used as a pictorial example to show children what their repeated addition looks like. By tilting or moving the arrays, children can see that there are two ways to represent each multiplication, for the same answer. This helps children to understand the commutative law.



5 x 2 = 10

2 x 5 = 10

Process through concrete – pictorial – abstract

Children should work with concrete objects first, use these alongside their pictorial representatives and then finally see concrete, with pictorial and then the abstract calculation or words. This helps children to make the connections. Children should not move between these stages until they are ready, and concrete resources should be made readily available for all children to support.

concrete – numicon

pictorial - dots

abstract - calculation

5 x 3 = 15

Calculation symbols

Children should be able to recognise, understand, read and write the x and = symbols. They should be able to use these to form their own calculations.

Explore with numbers

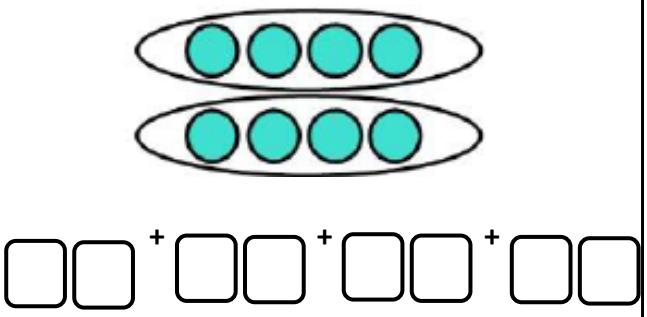
Children should be given the chance to explore with numbers.

E.g. If I make 5 towers of two blocks, how many blocks do I have?

If I take one tower away, how many do I have now?

Concrete → Pictorial → Abstract example:

4 x 2 = 8

Concrete:	Pictorial:	Abstract:
		2 x 4 = 8 4 x 2 = 8 2 + 2 + 2 + 2 = 8 4 + 4 = 8 2 lots of 4 are 8

Key vocabulary:

groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals

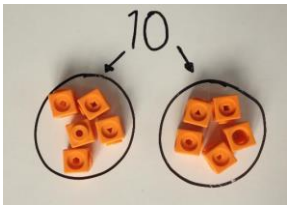
Key skills:

- Count in multiples of 2, 5 and 10 to 100.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays, with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens. Begin to understand doubling using concrete objects and pictorial representations.

Division:

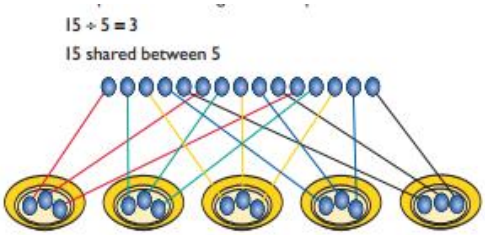
Sharing equally between groups

Children should understand that division requires sharing equally. Children should be encouraged to have a lot of practise at sharing between them and others using a range of different resources
e.g. Can you share these 10 cubes equally between the two of us?



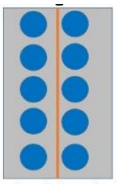
Combine groups of the same size

Divisions builds on children's knowledge of number, in that it has to be equal. Once children are able to make show sharing equally, they can find how many each.
E.g. When you shared the 15 equally, how many were on each plate? Are there the same amounts on each plate?



Relate to halving

Allow children to make the connection between halving and dividing by 2.
E.g. What is half of 10? 5. That's 10 divided by 2. That looks like this.
How can we say that another way?



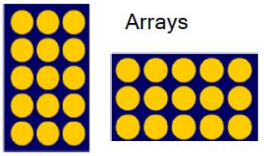
5 is half of 10.

Focus on 2s, 5s and 10s

Many of the examples given should be in groups of 2s, 5s and 10s, as highlighted in the curriculum. This is because children need to know these division facts by the end of year 2.
Children should be frequently counting in steps of 2, 5 and 10 in their maths starters in lessons.

Arrays

Arrays should be used as a pictorial example to show children how many groups make up the total. By tilting or moving the arrays, children can see that there are two ways to group the total amounts, making two different division calculations.

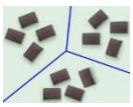


15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.

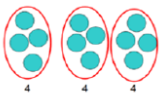
Process through concrete – pictorial – abstract

Children should work with concrete objects first, use these alongside their pictorial representatives and then finally see concrete, with pictorial and then the abstract calculation or words. This helps children to make the connections. Children should not move between these stages until they are ready, and concrete resources should be made readily available for all children to support.

concrete – blocks



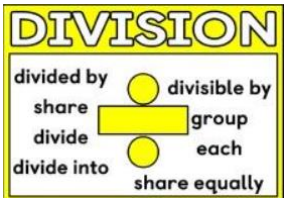
pictorial - dots



abstract - calculation
12 shared between 3 is 4.

Calculation symbols

Children should be able to recognise, understand, read and write the ÷ and = symbols. They should be able to use these to form their own calculations.

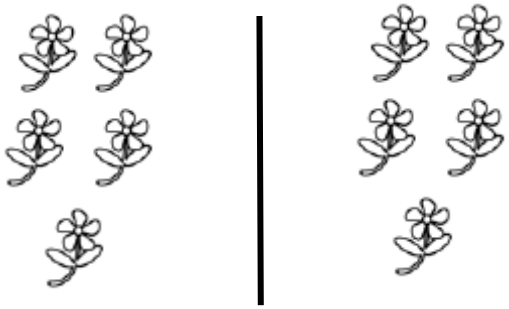


Explore with numbers

Children should be given the chance to explore with numbers.
E.g. If I make 5 towers of two blocks, how many blocks do I have?
If I take one tower away, how many do I have now?

Concrete → Pictorial → Abstract example:

10 ÷ 2 = 5

Concrete:	Pictorial:	Abstract:
		10 ÷ 2 = 5 10 ÷ 5 = 2 10 shared between 2 is 5

Key vocabulary:

share, share equally, __ each, groups, groups of, lots of, array, calculation, amount

Key skills:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations, arrays, with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand division, and finding simple fractions of objects, numbers and quantities. ($^1/_2$, $^1/_4$)
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year Two

Addition:

Mental strategies

Develop strategies for mental calculations (as listed below) so children can use these strategies to build fluency, their arithmetic skills and use them to establish formal written methods.

Adding 3 single digit numbers

When adding up 3 numbers, children should be taught to look for number bonds to 10 first of all. They should also look for doubles patterns. Doubles patterns up to 10 + 10 should have already been established in Year 1, but continued practise in Year 2 will help this become an embedded strategy for these children.

Number bonds:
8 + 5 + 2 → 8 + 2 = 10 10 + 5 = 15.

Doubles first:
6 + 3 + 6 → 6 + 6 = 12 12 + 3 = 15.

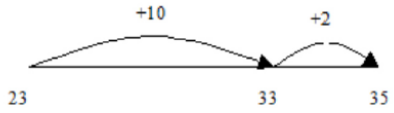
Partitioning into tens and ones

Children should be using concrete objects (dienes) to partition two-digit numbers into tens and ones. Once children are familiar with using the dienes, they should move on to drawing this pictorially. Knowing this will help with formal calculations later in the school year.

concrete (47) pictorial

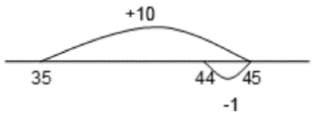
Counting in tens and ones

Children should be developing their mental calculation strategies by using their understanding of partitioning to count on in tens first and then ones. This can be done effectively by drawing dienes or using a number line.



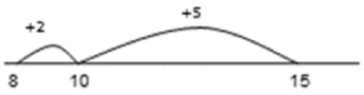
Adding 9 or 11 by adjusting by 1

Children should be able to add 9 by adding on ten and subtracting one. They should be able to add 11 by adding on ten and adding another one.



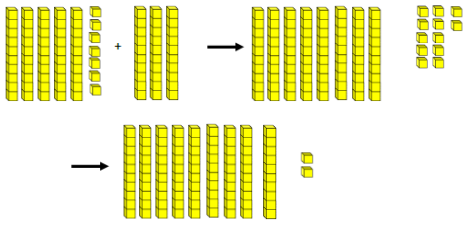
Bridging through 10

When adding numbers together that bridge through ten, children should consider adding to make 10 first, and then adding on the rest.
E.g. 8 + 7 → 8 + 2 = 10 10 + 5 = 15.
This should be done using the CPA (concrete – pictorial – abstract) method.



Exchanging

When bridging through ten by adding together larger numbers, children will need to see that ten ones are equal to one ten. This can be shown by adding using dienes and exchanging ten ones for one ten. Making 57 + 35 = 80 + 12 = 92.



Expanded written methods

47 + 25 → 40 + 7 + 20 + 5 →
40 + 20 + 7 + 5 = 60 + 12 → 60 + 12 = 72 (60 + 10 + 2)

40 + 7
+ 20 + 5
60 + 12 = 72

Formal written methods

Once true understanding of tens and ones has been established, use this method. Always use place value headings to show the value of the numbers.

56 + 13 →

T	O
5	6 +
1	3
= 6	9

Missing number calculations

Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 34 = 67 - 33 12 – 9 = 14 - 11

All possibilities of missing numbers shown to children to they understand how these numbers connect.

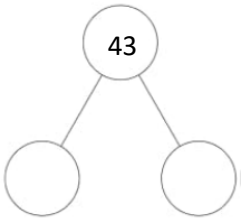
E.g. 29 - 14 = [] [] = 29 - 14 14 + [] = 29 29 = [] + 14

Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.

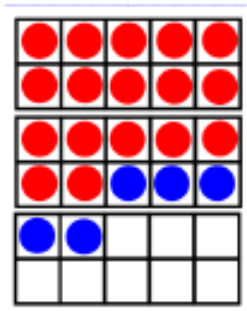
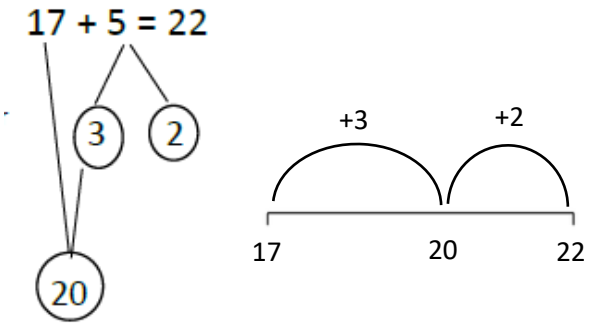
E.g. two numbers add up to make 43. What could those two numbers be?

How many different ways can you find?



Concrete → Pictorial → Abstract example:

17 + 5 = 22

Concrete:	Pictorial:	Abstract:
		17 + 5 = 22 Explore related facts: 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17

Key vocabulary:

(Y1) add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation

(Y2) sum, tens, ones, partition, units, addition, column, exchange, number line

Key skills:

- Add a 2-digit number and ones (e.g. 27 + 6).
- Add a 2-digit number and tens (e.g. 23 + 40).
- Add pairs of 2-digit numbers (e.g. 35 + 47).
- Add three single-digit numbers (e.g. 5 + 9 + 7).
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.).
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones).
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Subtraction:

Count back in tens and ones

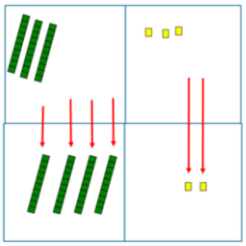
Children should be regularly practising counting back in tens and in ones, from any given number.

E.g. 32 → 31, 30, 29, 28

89 → 79, 69, 59, 49

This will help them to subtract by partitioning.

75 - 42 =



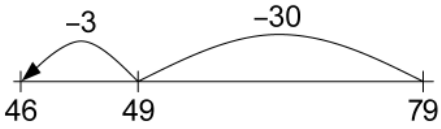
70	5
-40	2
30	3

Partitioning into tens and ones

Subtract by portioning the numbers into tens and ones first. This will allow the children to take away the tens first, and then ones for an accurate answer and avoid counting back in ones when using larger numbers.

From dienes to number lines

Once children have established how to subtract by portioning using dienes (concrete), it is important to move them on to a more efficient pictorial method, such as a number line.

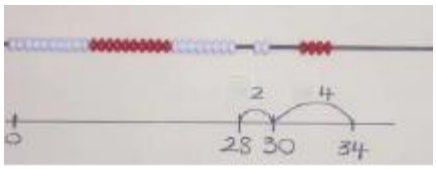


Making 10

Another strategy to show children is to make make it to the next ten and count on (or backwards) from there. This works well when the numbers are close together in value.

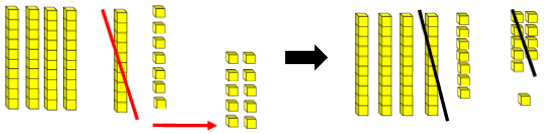
E.g. 34 – 28 → 28 + 2 = 30 + 4 = 34 or 34 – 4 = 30 – 2 = 28

→ both times a difference of 6.



Regrouping (Expanded written methods)

Also known as *exchanging*. Once children are familiar with subtracting by partitioning and using a number line, they will need to be able to use these techniques when the number of ones being taken away is greater than the number of ones we have started with.



E.g. 57 – 19 → 5 tens + 7 ones = 4 tens and 17 ones. Subtract 1 ten and 9 ones = 3 tens and 8 ones = 38.

Formal written methods

Once true understanding of tens and ones has been established, use this method. Always use place value headings to show the value of the numbers.

T	O
7	5
-4	2
3	3

Missing number calculations

Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 34 = 67 - 33 12 – 9 = 14 - 11

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 29 - 14 = [] [] = 29 - 14 14 + [] = 29 29 = [] + 14

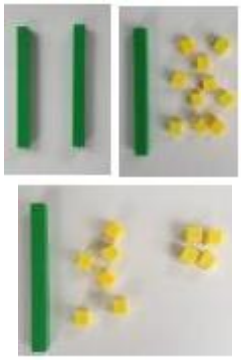
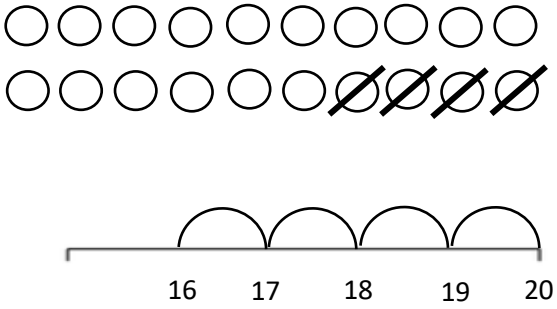
Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.

E.g. The difference between my two numbers is 24. My first number is three times larger than the smaller number. What are the two numbers? (Use dienes for support)

Concrete → Pictorial → Abstract example:

20 – 4 = 16

Concrete:	Pictorial:	Abstract:
		20 - 4 = 16 Explore related facts: 20 – 4 = 16 20 – 16 = 4 16 + 4 = 20 4 + 16 = 20

Key vocabulary:

(Y1) equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation
(Y2) difference, count on, strategy, partition, regroup, exchange, tens, ones

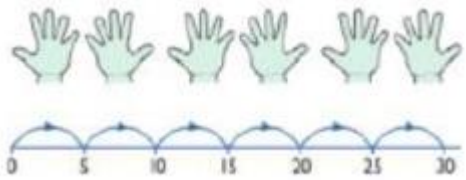
Key skills:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Multiplication:

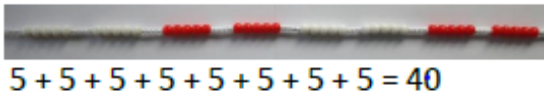
Count in multiples of 2, 3, 4, 5 and 10

Children should regularly practise counting in multiples of 2, 3, 4, 5, and 10 from 0. This needs to be embedded before children can be taught to multiply using calculations.



Repeated addition

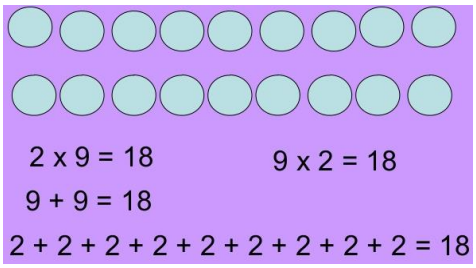
Multiplication builds on children's knowledge of addition, in that it is repeated addition. For instance, when looking at 8 x 5, children can draw upon their knowledge of counting in 5s and see that 8 x 5 is the same as adding on 5 each time.



5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40

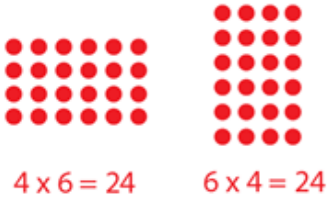
Relate to doubling

Allow children to make the connection between doubling and multiplying by 2._E.g. What is double 9? 18.
That's 9 lots of 2s. That looks like this.
How can we say that another way?



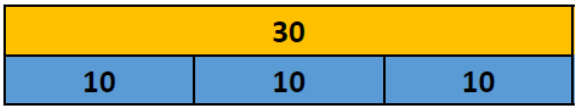
Arrays

Arrays should be used as a pictorial example to show children what their repeated addition looks like. By tilting or moving the arrays, children can see that there are two ways to represent each multiplication, for the same answer. This helps children to understand the commutative law. These should include all multiples, not just the 2x, 5x and 10x expected that children are expected to learn by the end of Year 2.



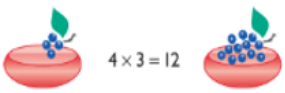
Bar model

Following on from arrays, bar models should be used to help children visual the concept of multiplication and see it as equal groups of the same number/amount to make a total.



Express as a calculation

Children should be able to recognise, understand, read and write the x and = symbols. They should be able to use these to form their own calculations.



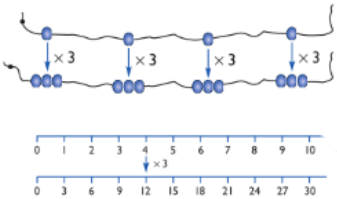
Missing number problems

Using understanding of the inverse and practical resources to solve missing number problems. Children need to practise this alongside division as well to know how to find the inverse.

7 x 2 = □ □ = 2 x 7 7 x □ = 14 14 = □ x 7

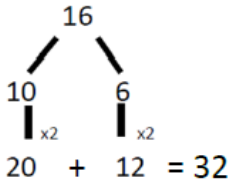
Develop the ideas of scaling

Begin to develop the idea that multiplication can be used to scale something up e.g. make it 3 times bigger/taller/heavier.



Work towards written methods

Begin to look at multiplying (mostly doubling) larger numbers by partitioning into tens and ones, multiplying and adding. Follow this example:



Explore with numbers

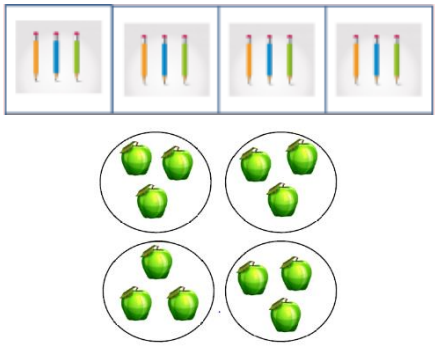
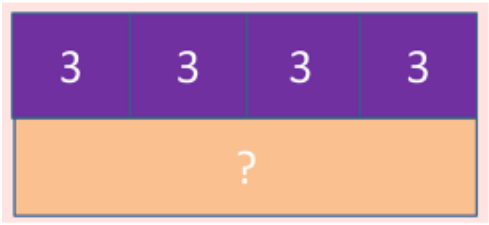
Allow children to solve more open problems e.g.

The answer is 12. Show as many different arrays as you can that make 12.

What two numbers could have been multiplied to make 12? Find all possibilities.

Concrete → Pictorial → Abstract example:

3 x 4 = 12

Concrete:	Pictorial:	Abstract:
		3 x 4 = 12 3 + 3 + 3 + 3 = 12 Explore related facts: 3 x 4 = 12 4 x 3 = 12 12 ÷ 4 = 3 12 ÷ 3 = 4

Key vocabulary:

(Y1) groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals

(Y2) times, multiplication, inverse, operation, equal groups, times as big as, column, row

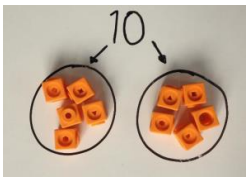
Key skills:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Division:

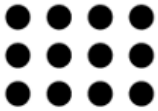
Halving

Children should be building on knowledge from Year 1 that division means sharing between equal groups. This links well to halving and children need to know that halving is the same as dividing into two equal groups.



Arrays

Arrays should be used as a pictorial example to show children how many groups make up the total. Children should use arrays to find all calculations for multiplication and division by looking at grouping by rows and by columns.



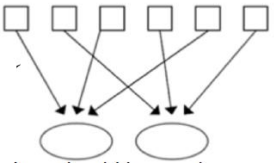
$3 \times 4 = 12$
$4 \times 3 = 12$
$12 \div 3 = 4$
$12 \div 4 = 3$

Understand sharing and grouping

Children should be able to see division calculations as both sharing and grouping. Sharing meaning the total is shared out between a certain number of people, how many will they receive each? Grouping meaning the total is grouped so that everyone has a certain number each, how many people have that amount now?

Sharing:

6 sweets are shared between 2 people. How many sweets do they each get?

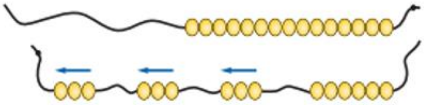


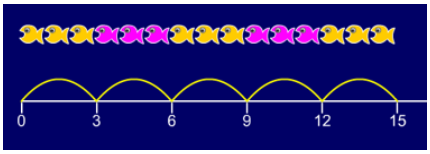
Grouping:

There are 6 sweets, how many people can have 2 sweets each?

Grouping using a number line

Use a number line to show grouping.
E.g. Group from zero in jumps of the divisor to find out 'how many groups of 3 are there in 15?





Using concrete objects and pictorial representatives

Continue using concrete objects to physically share and group to begin with before moving on to picture representations and eventually abstract calculations and word problems.

Repeated subtraction

Children should also see the connection between division and repeated subtraction, as they did between multiplication and repeated addition. It can be used to find unknown facts.
e.g. knowing $10 \div 2 = 5$ but not knowing $8 \div 2$ yet – it's one group of 2 less so $5 - 1 = 4$.

Calculation symbols

Children should be able to recognise, understand, read and write the ÷ and = symbols.
They should be able to use these to form their own calculations.

Missing number problems

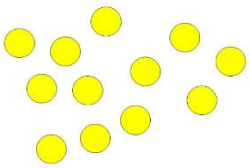
Children should be able to progress to find missing numbers in a variety of different division calculations.

$6 \div 2 = \square$ $\quad \square = 6 \div 2$ $\quad 6 \div \square = 3$ $\quad 3 = 6 \div \square$

$\square \div 2 = 3$ $\quad 3 = \square \div 2$ $\quad \square \div 2 = 3$ $\quad 3 = \square \div 2$

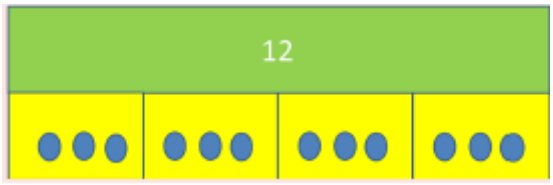
Explore with numbers

Children should be given the chance to explore with numbers
E.g. 12 ÷ [ ] = [ ] ← Find all possibilities for this.



Concrete → Pictorial → Abstract example:

12 ÷ 4 = 3

Concrete:	Pictorial:	Abstract:
		12 ÷ 4 = 3 12 ÷ 3 = 4 3 x 4 = 12 4 x 3 = 12 12 shared between 4 is 3

Key vocabulary:

(Y1) share, share equally, __ each, groups, groups of, lots of, array, calculation, amount
(Y2) divide, divided by, divided into, division, grouping, number line, left, left over

Key skills:

- Count in steps of 2, 3 and 5 from 0.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year Three

Addition:

Mental strategies

Develop strategies for mental calculations to allow children to use these strategies to build fluency, their arithmetic skills and use them to establish formal written methods. In Year 3, children should be encourage to count on in ones, tens and hundreds mentally from a given number.

E.g. 24 → 25, 26, 27, 28 34, 44, 54, 64 124, 224, 324, 424

Partitioning into tens and ones

At first, children should be taught to partition both numbers and recombine.

E.g. 247 + 125 → 200 + 100 = 300 → 300 + 60 + 12 = 372

40 + 20 = 60

7 + 5 = 12

Then, move on to count on by partitioning the second number only.

E.g. 247 + 125 → 247 + 100 + 20 + 5

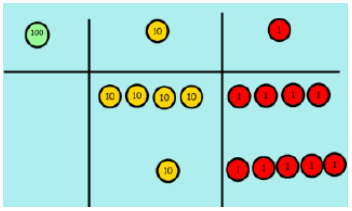
= 347 + 20 + 5

= 367 + 5

= 372

Model with manipulatives

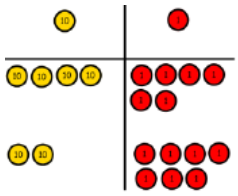
(44 + 15 =)



This should be done using manipulatives first, such as place value counters and dienes. Then it should move on to a written methods. Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Concrete and pictorial addition methods

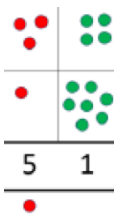
Children should be shown how to add larger numbers using concrete manipulatives (such as place value counters and a place value chart), then using pictorial methods (such as drawing these counters in their books) so that we can establish how well the children understand the idea of adding hundreds, tens and ones, as well as carrying over, before looking at written methods.



Modelling regrouping

When regrouping occurs, children need to physically see how, for example, 13 ones become 1 ten and 3 ones by exchanging the place value counters or dienes.

They should the practise writing this down.



4 ones + 7 ones = 11 ones.

11 ones = 1 ten + 1 one.

1 ten is carried over.

This layout prepares the children well for column addition when they are ready.

Formal written methods

Use formal written methods regularly, assuring that children understand and discuss the **value** of the numbers

e.g. 345 + 56

H T O

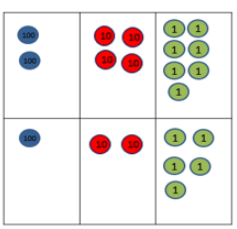
3 4 5 +

5 6

5 ones + 6 ones = 11 ones = 1 ten and 1 one

4 tens + 5 tens + 1 ten (carried over) = 10 tens – same as 1 hundred.

Remind children and use vocab at all times, so it is not 4 + 5+ 1



200 + 40 + 7

100 + 20 + 5

300 + 60 +12 = 372

247

+125

12

60

300

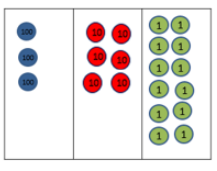
372

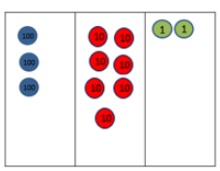
247

+125

372

10





but 4 tens plus 5 tens plus 1 ten – how many tens? What is this the same as?
Use dienes, base 10, place value counters and other resources to help show children these equivalents.
Remind children that **number lines** are also really valuable for adding on in hundred, tens and ones (see Year 2).

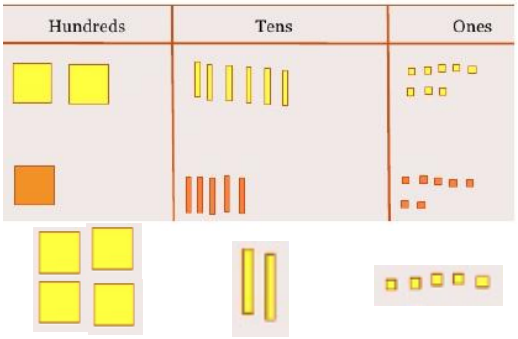
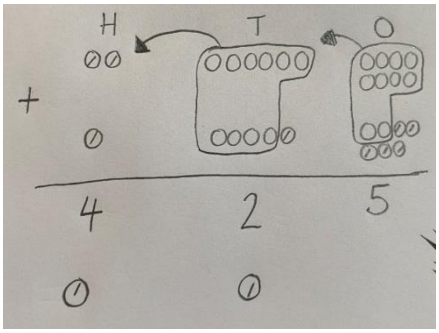
Missing number calculations

Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 420 = 314 +106 605 + 230 = 630 + 205
All possibilities of missing numbers shown to children to they understand how these numbers connect.
E.g. 150 - 35 = [] [] = 150 - 35 35 + [] = 150 150 = [] + 35

Explore with numbers
Give children chance to solve problems or find lots of solutions to one problem.
E.g. 1 leather bag and 2 pairs of Nike trainers add up to £340.
Both the bag and the trainers cost more than £80 each.
How much could the leather bag and the Nike shoes cost? Find all possibilities.

Concrete → Pictorial → Abstract example:

268 + 157 = 425

Concrete:	Pictorial:	Abstract:
Using dienes 		268 + 157 = 425 Explore related facts: 268 + 157 = 425 157 + 268 = 425 425 – 268 = 157 425 – 157 = 268 What is the total of 268 and 157?

Key vocabulary:

(Y1) add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation
(Y2) sum, tens, ones, partition, units, addition, column, exchange, number line
(Y3) tens boundary, hundreds boundary, carry over, increase, vertical, expanded, inverse

Key skills:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, including those with a total exceeding 100.
- Add a three-digit number and ones mentally (175 + 8).
- Add a three-digit number and tens mentally (249 + 50).
- Add a three-digit number and hundreds mentally (381 + 400).
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones).
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Subtraction:

Mental strategies

Develop strategies for mental calculations to allow children to use these strategies to build fluency, their arithmetic skills and use them to establish formal written methods. In Year 3, children should be encourage to count back in ones, tens and hundreds mentally from a given number.

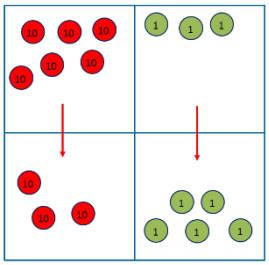
E.g. 650 → 649, 648, 647, 646 640, 630, 620, 610 550, 450, 350, 250

Subtracting with no exchanging

Allow children to use place value counters and dienes to subtract in hundreds, tens and ones in columns with no exchanging.

By partitioning the number and keeping the subtraction in columns, it best prepares the children for their written column method later on.

Encourage children to write the calculation alongside the concrete example shown to see the correlation.

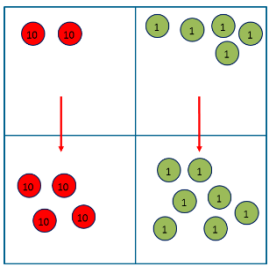


$$\begin{array}{r} 90\ 8 \\ -\ 30\ 5 \\ \hline 60\ 3 \end{array}$$

Subtracting with exchanging

Only once children are secure with subtracting with no exchanging, should they be introduced to the idea of exchanging ten for 10 ones, or hundred for 10 tens. Using this vocabulary each time this is modelled will help children understand what is being exchanged and why.

Make them physically exchanging the 10 counter (or dienes ten stick) for 10 ones to they can see an exchange taking place – but the value staying the same. Again, write the calculation alongside the concrete example.



$$\begin{array}{r} {\scriptstyle 60}\quad {\scriptstyle 1} \\ 7\not{0}\ \ {}^{1}2 \\ -\ 40\ \ 7 \\ \hline 20\ \ 5 \end{array}$$

Written methods

Once children are secure with concrete examples and pictorial representations, they should be shown how to use column subtraction as the most efficient method. This requires a clear understanding of place value, especially when needing to exchange. The presentation for this is key, so a lot of modelling is required to ensure children understand that the numbers need to line up, and why.

H	T	O
3	6	8 -
1	2	8
2	4	0

Continuing with partitioning methods and number lines may help ease the transition into column method.

Missing number calculations

Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 145 = 200 - 55 150 – 95 = 100 - 45

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 124 - 65 = [ ] [ ] = 124 - 65 65 + [ ] = 124 124 = [ ] + 65

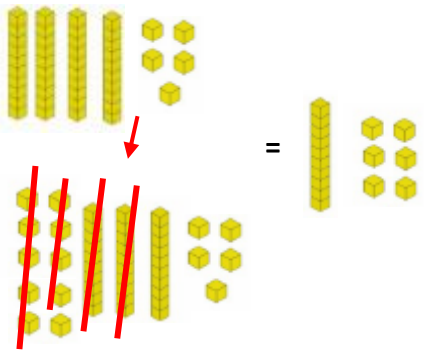
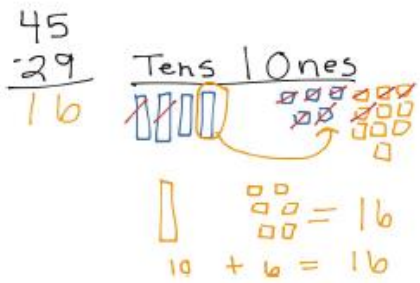
Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.

E.g. [ ] – [ ] = 124 The first number is less than 200.
Find 5 different ways to complete this calculation.

Concrete → Pictorial → Abstract example:

45 – 29 = 16

Concrete:	Pictorial:	Abstract:
		45 - 29 = 16 Explore related facts: 45 – 29 = 16 16 + 29 = 45 45 – 16 = 29 29 + 16 = 45 Find the difference between 45 and 29.

Key vocabulary:

(Y1) equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation
(Y2) difference, count on, strategy, partition, regroup, exchange, tens, ones
(Y3) decrease, inverse, digit, value, method, hundreds

Key skills:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above).
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

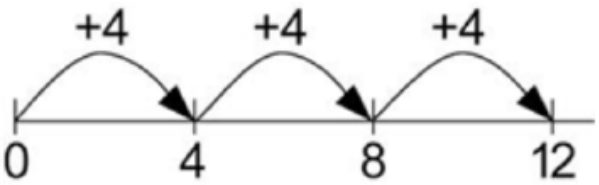
Multiplication:

Mental strategies

Children should rely on known facts and counting to help them with their multiplication.
Known facts include doubles, multiplying by 10 and counting in multiples of 4, 8, 50 and 100.
This should be practised often so children can rely on it during more complex calculations.

Repeated addition

Children need to see the link between repeated addition and multiplication. For instance 3 x 40 → 40, 80, 120.
Relies on them using repeated addition (counting up) and uses their multiples knowledge.
It may help for children to see this written down as well.

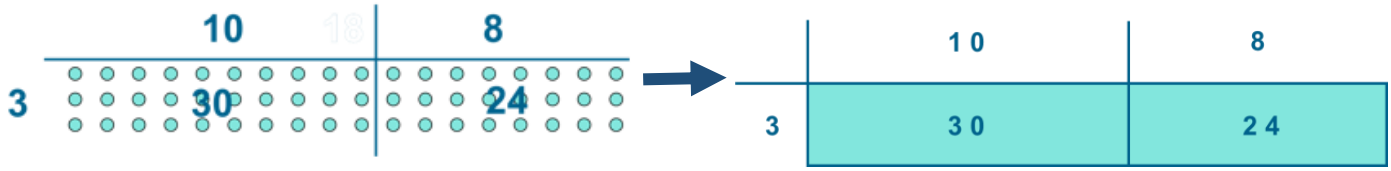


Times table recall

Children are expected to know their 1x, 2x, 3x, 4x, 5x, 8x and 10x by the time they reach the end of Year 3.
In order to be able to fluently recall these tables, they need to know and remember the facts, rather than spend the time counting up or drawing a diagram to help them. However, children will need to spend time looking at the facts alongside diagrams, repeating out loud, using songs and so on to learn them efficiently. Knowing these times table facts should also include divisions. Children can get practise with these tables by using Times Table Rock Stars.

Grid method

Use arrays to initially show how the grid method works.
The idea is this provides a visual for the children to see how multiplication is done in 'lots of' or 'groups of'.
For *3 x 18*:
Partition 18 into 10 and 8. The single digit goes to the left and the 2-digit number is partitioned on top.



This method relies on the children having a solid knowledge of place value, partitioning into tens and ones and counting in multiples. This can be shown with dienes, counters or cubes to start with and then progress onto a pictorial representative. After that, an abstract approach (as shown on the right)

Multiples of ten

In order to help with grid method, partitioning method and eventually expanded column method, children need to continually practise multiplying in multiples of 10.
e.g. If we know 4 x 5 = 20, we know that 40 x 5 = 200. 40 is ten times bigger, so the answer becomes ten times bigger. This will need to be shown using manipulatives and diagrams for a better understanding.

Expanded column method

10 + 3
x 4
12 (3 x 4)
+ 40 (10 x 4)
52

Partition 13 into 10 + 3 then multiply each number by 4. Add the partial products (12 and 40) together.

This can be introduced to GD children:

13 x 4 = 52

(10 x 4) + (3 x 4) = 40 + 12 = 52

Missing number problems

Using understanding of the inverse and practical resources to solve missing number problems.
Children need to practise this alongside division as well to know how to find the inverse.

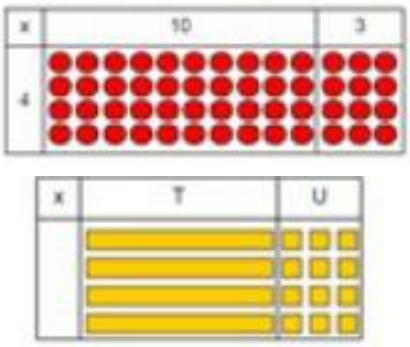
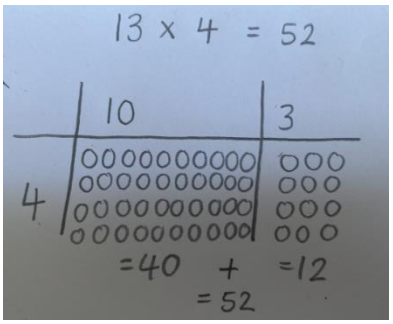
8 x 3 = □ □ = 3 x 8 3 x □ = 24 24 = □ x 8

Explore with numbers

Allow children to solve more complicated problems and explore number using what they know.
E.g. [ ] 6 x 7 = 3 9 2 Find the missing number.

Concrete → Pictorial → Abstract example:

3 x 4 = 12

Concrete:	Pictorial:	Abstract:
		13 x 4 = 52 13 + 13 + 13 + 13 = 52 Explore related facts: 13 x 4 = 52 4 x 13 = 52 52 ÷ 4 = 13 52 ÷ 13 = 4

Key vocabulary:

(Y1) groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals
(Y2) times, multiplication, inverse, operation, equal groups, times as big as, column, row
(Y3) partition, grid method, multiple, product, tens, ones, 2-digit number, value

Key skills:

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single-digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutative law e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240.
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems and for missing number problems.

Division:

Mental strategies

Children should use known facts to help them to calculate TO ÷ O calculations.
They may use their knowledge of multiples of 4, 8, 50 and 100 to answer division problems such as
72 ÷ 8 → 8, 16, 24, 32, 40, 48, 56, 64, 72 → 9 lots of 8.
They may partition the number into manageable chunks e.g. 39 ÷ 3 → 30 ÷ 3 = **10**, 9 ÷ 3 = **3** so 39 ÷ 3 = **13**

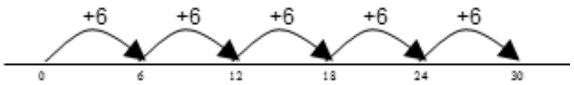
Division facts

Once children have learned their times table facts for 1x, 2x, 3x, 4x, 5x, 8x and 10x, they should be able to use these facts to answer corresponding division facts through recall.
E.g. I know that 5 x 7 = 35, so I should also recognise the answer to 35 ÷ 7 and 35 ÷ 5.

Grouping

Children should understand that dividing means the total shared into equal groups.
By grouping, the children can see how many are needed to make the total using concrete and pictorial representatives.
e.g. How many 6s are in 30?

30 ÷ 6 can be modelled as:



This shows 5 jumps of 6, so 30 ÷ 6 = 5

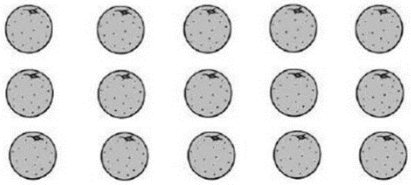
Becoming more efficient using a number line

10 groups 2 groups

Children need to be able to partition the dividend in different ways.

Arrays

Using arrays to show grouping in rows and columns can help children to physically see and move the groups when using concrete objects, and draw around and count when using a pictorial representative.
This is also a huge benefit for introducing factors and calculation families.
It also helps them to physically make groups and arrays when beginning to look at division calculation with remainders.



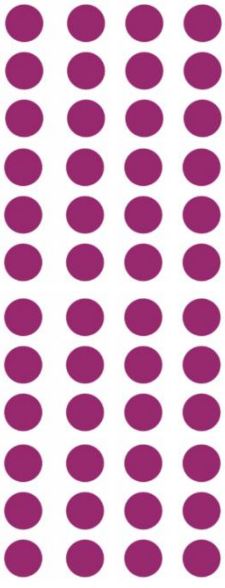
Remainders

Use a combination of arrays and number lines to show what remainders are and how we know when there is a remainder.
e.g. **49 ÷ 4 = 12 r1**

Sharing: 49 shared between 4. How many left over?

Grouping: How many 4s make 49? How many are left over?

What is 49 divided by 4? What is the remainder?



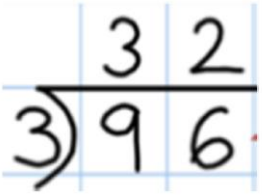
12 groups of 4.

With 1 remainder.

Short division

Greater depth children can slowly be introduced to a written division method. The child would need to be confident with their times table knowledge and understand how the dividend (96) is shared by the divisor (3). Don't use this methods with reminders just yet.

Short division: Limit numbers to NO remainders in the answer OR carried (each number must be a multiple of the divisor)



Missing number problems

Children should be able to progress to find missing numbers in a variety of different division calculations.

45 ÷ 9 = □ | □ = 45 ÷ 9 | 45 ÷ □ =9 | 9 = 45 ÷ □

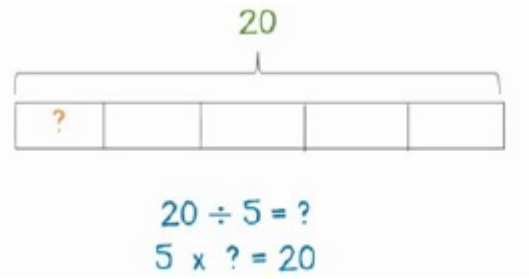
□ ÷ 9 = 5 | 9 = □ ÷ 5 | □ ÷ 5 = 9 | 5 = □ ÷ 9

Explore with numbers

Children should be given the chance to explore with numbers
E.g. 72 ÷ [ ] = [ ] ← Find all possibilities for this.

Concrete → Pictorial → Abstract example:

20 ÷ 5 = 4

Concrete:	Pictorial:	Abstract:
		20 ÷ 5 = 4 20 ÷ 4 = 5 4 x 5 = 20 5 x 4 = 20 20 shared equally between 5 is 4.

Key vocabulary:

(Y1) share, share equally, __ each, groups, groups of, lots of, array, calculation, amount
(Y2) divide, divided by, divided into, division, grouping, number line, left, left over
(Y3) inverse, short division, carry, remainder, multiple, dividend, divisor

Key skills:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 =6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Year Four

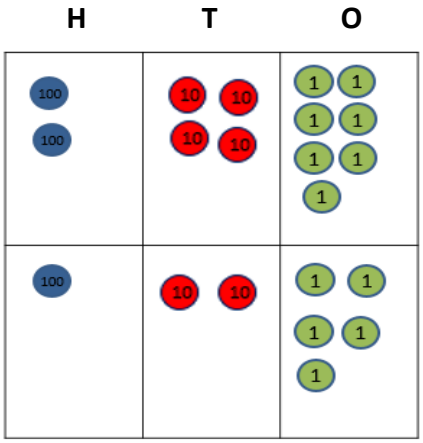
Addition:

Mental strategies

Children should still be using mental strategies built up over the years, such as doubles, near doubles (6 + 7), counting on in ones, tens, hundreds and thousands from any given number and number bonds to 10, 20, 50 and 100.

Expanded column method

Following on from what they learned in Year 3, children should start with an expanded column method using place value counters or dienes.
This should then be shown by looking at adding up the hundreds, tens and ones separately.
This will then prepare them nicely into seeing these numbers being adding up vertically in column method.
Always use place value headings to help children understand the values of the digits.

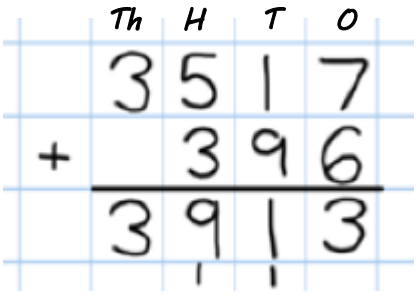


200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

247
+125
12
60
300
372

Formal column method

When children are ready, they need to progress on to the formal column method. This requires learning the reasoning behind the method and how it relates to the previous methods they have learned. It may be worth showing the expanded methods alongside the formal method to start with.
Rather than 'start with the ones', use 'start with the lowest value', so it isn't confusing when the children progress to working with decimals.
Re-inforce the language used when adding and carrying over e.g. '7 ones add 6 ones make 13 ones. 13 ones is the same as 1 ten and 3 ones, so the ten is carried over into the tens column.'
The same applies for 5 hundred + 3 hundreds make 8 hundreds (not 5 + 3 = 8).
Place value grids are really important to use in the early stages of column addition.



Understanding the value of the digits

For all column addition, whether working with larger numbers, decimals or both, it is important that children know the value of the numbers they are adding, and are using that language in their explanations and through modelling in lessons.
E.g. "*one ten add 9 tens equals 10 tens. Plus the carried over ten makes 11 tens. That is the same as 1 hundred and 1 ten. So 1 goes in the tens column and the hundred is carried over to the hundreds column.*"
This is really important when working with measures, such as adding money. Children need to be familiar with 10 hundredths making 1 tenth, and 10 tenths making one.

Choosing the best method

It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children.
E.g. 99 + 47 → Much better to take 1 from the 47 to make 100 + 46 = 146 than to use column method.

Missing number calculations

Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 1,583 = 1,097 + 486 6,709 + 1,495 = 3,467 + 4,737

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 1,583 – 486 = [] [] = 1,583 - 486 486 + [] = 1,583 1,583 = [] + 486

Explore with numbers

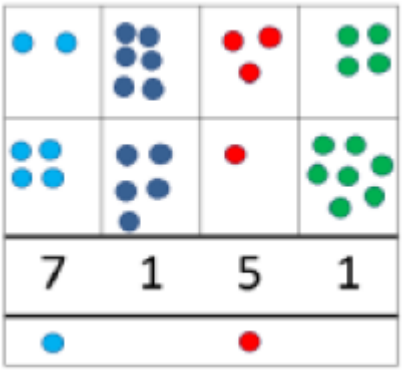
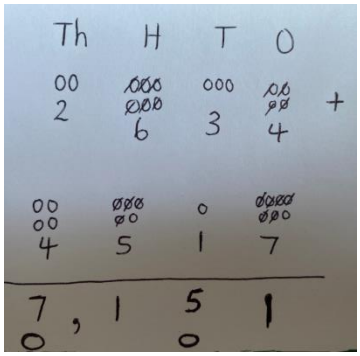
Give children chance to solve problems or find lots of solutions to one problem.

E.g. I buy some bananas and pears. Bananas cost 43p each, pears cost 34p each.

I spend exactly £3.94. Exactly how many bananas and pears did I buy?

Concrete → Pictorial → Abstract example:

2634 + 4517 = 7151

Concrete:	Pictorial:	Abstract:
		Th H T O 2 6 3 4 + 4 5 1 7 7 1 5 1 1 1

Key vocabulary:

(Y1) add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation

(Y2) sum, tens, ones, partition, units, addition, column, exchange, number line

(Y3) tens boundary, hundreds boundary, carry over, increase, vertical, expanded, inverse

(Y4) compact, thousands, tenths, hundredths, digits, value

Key skills:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition.
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

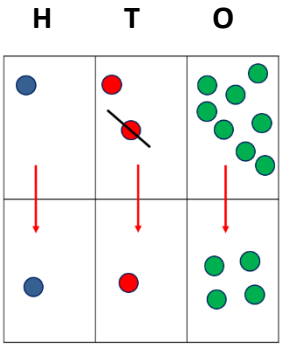
Subtraction:

Mental strategies

Children should still be using mental strategies built up over the years, such as halving, counting back in ones, tens, hundreds and thousands from any given number and number bonds to 10, 20, 50 and 100.

Expanded column method

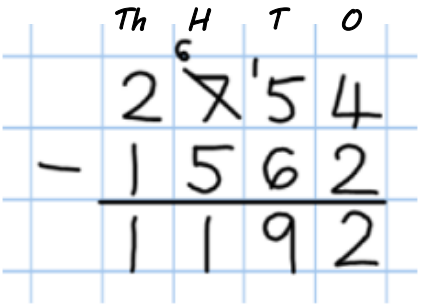
Following on from what they learned in Year 3, children should start with an expanded column method using place value counters or dienes. This should then be shown by looking at subtracting the hundreds, tens and ones separately (when no exchanging is required first, then introduce exchanging) This will then prepare them nicely into seeing these numbers being subtracted vertically in column method. Always use place value headings to help children understand the values of the digits.



$$\begin{array}{rrrr} & & \overset{20}{\cancel{30}} & \overset{1}{2} \\ - & 100 & 10 & 4 \\ \hline & 100 & 10 & 8 \end{array}$$

Formal column method

When children are ready, they need to progress on to the formal column method. This requires learning the reasoning behind the method and how it relates to the previous methods they have learned. It may be worth showing the expanded methods alongside the formal method to start with. Rather than 'start with the ones', use 'start with the lowest value', so it isn't confusing when the children progress to working with decimals. Re-inforce the language used when subtracting and exchanging over e.g. '3 ones minus 6 ones, oh we need to exchange one of the tens for ten ones to make this 13 ones so we can subtract So 5 tens become 4 tens and 3 ones becomes 13 ones. 13 ones minus 6 ones is 7 ones.' The same applies for 5 hundred - 3 hundreds make 2 hundreds (not 5 - 3 = 2). Place value grids are really important to use in the early stages of column subtraction.



Understanding the value of the digits

For all column subtraction, whether working with larger numbers, decimals or both, it is important that children know the value of the numbers they are subtracting, and are using that language in their explanations and through modelling in lessons.

E.g. "*7 tens subtract 9 tens cannot be done without going into minus numbers. We need to exchange one of the hundreds for 10 tens, to give 17 tens in total. So 3 hundreds become 2 hundreds and we exchange that hundred for ten tens, making 17 tens. 17 tens subtract 9 tens then equals 8 tens.*"

This is really important when working with measures, such as adding money. Children need to be familiar with one also equalling 10 tenths and one tenth also equalling 10 hundredths.

Choosing the best method

It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children.

E.g. 103 – 86 → Much better to count up from 86 to the 103 than do lots of exchanging in a column method. 86 + 4 = 90 + 10 = 100 + 3 = 103 10 + 4 + 3 = 17.

Missing number calculations

Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 4,827 = 9,873 – 4,046 2,847 - 1,744 = 5,246 – 4,143

All possibilities of missing numbers shown to children to they understand how these numbers connect.

E.g. 1,583 – 486 = [] [] = 1,583 - 486 486 + [] = 1,583 1,583 = [] + 486

Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.

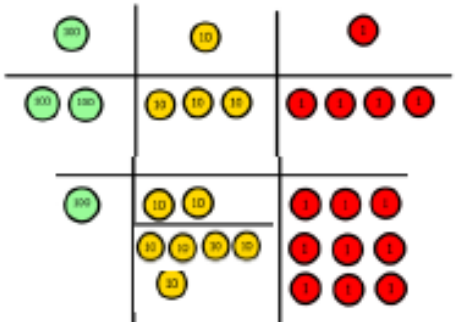
E.g. The difference in cost for a book and a pen is £1.25.

How much could the book and the pen cost? Find 5 different possibilities.

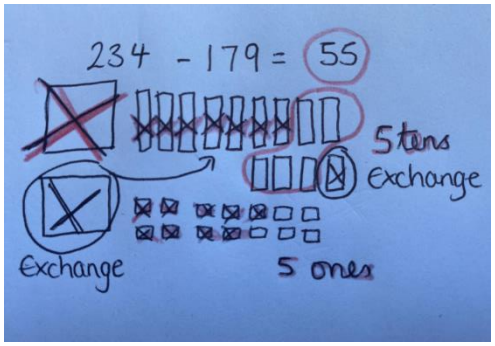
Concrete → Pictorial → Abstract example:

234 – 179 = 55

Concrete:



Pictorial:



Abstract:

	H	T	O
	1	12	
	~~2~~	~~3~~	14
+	1	7	9
		5	5

Key vocabulary:

(Y1) equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation

(Y2) difference, count on, strategy, partition, regroup, exchange, tens, ones

(Y3) decrease, inverse, digit, value, method, hundreds

(Y4) thousands, tenths, hundredths, decimal

Key skills:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Multiplication:

Mental strategies

Children should rely on known facts and counting to help them with their multiplication.
Known facts include doubles, multiplying by 10 and counting in multiples of 6, 7, 9, 25 and 1000.
This should be practised often so children can rely on it during more complex calculations.

Times table recall

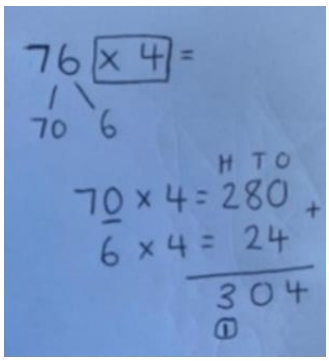
Children are expected to know all of their times table and division facts up to 12 x 12 by the time they reach the end of Year 4. In order to be able to fluently recall these tables, they need to know and remember the facts, rather than spend the time counting up or drawing a diagram to help them. However, children will need to spend time looking at the facts alongside diagrams, repeating out loud, using songs and so on to learn them efficiently. Knowing these times table facts should also include divisions. Children can get practise with these tables by using Times Table Rock Stars.

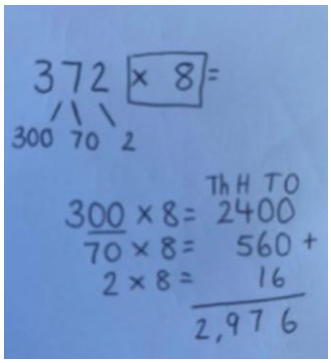
Multiples of ten/hundred

In order to help with partitioning method and eventually column method, children need to continually practise multiplying in multiples of 10 and 100
e.g. If we know 4 x 5 = 20, we know that 40 x 5 = 200. 40 is ten times bigger, so the answer becomes ten times bigger. We also know that 400 x 5 = 2,000. This will need to be shown using manipulatives and diagrams for a better understanding.

Partitioning

Start introducing multiplying a 2 digit or 3 digit number by a single digit by using partitioning, as this leads on nicely from the grid method taught in Year 3.
Set it out as shown here →
By putting the box around the single digit, it helps children to remember to multiply the HTO by that number, and avoid misconceptions such as 76 → 70 x 6.
Use place value headings with the answers so they can be added using column addition there and then.





Column multiplication

Once children feel confident with partitioning, they can progress on to using the column method to multiply. This relies on their place value knowledge, their times table knowledge and being able to calculate and keep a lot of information in their heads before calculating.
Children should begin this method alongside partitioning to feel familiar and check their working.

Th	H	T	O	
	1			
	4	5	2	x
			3	
1	3	5	6	

Approximating and estimating to minimise error

When looking at a calculation, children should be able to estimate what the answer should be close to before calculating. This involves making an approximation e.g. 346 x 9 → 350 x 10 = 3,500.

They should record this approximation to check their final answer against. This will help children to see if they are out by powers of 10. For instance, an answer of 314 would definitely have an errors in it.

Missing number problems

Using understanding of the inverse and practical resources to solve missing number problems.

Children need to practise this alongside division as well to know how to find the inverse.

20 x 3 = □ □ = 3 x 20 3 x □ = 60 60 = □ x 20

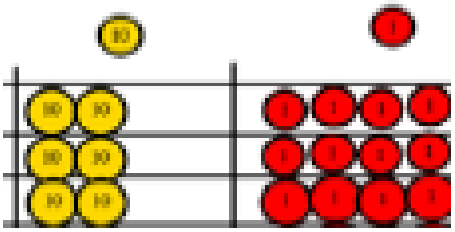
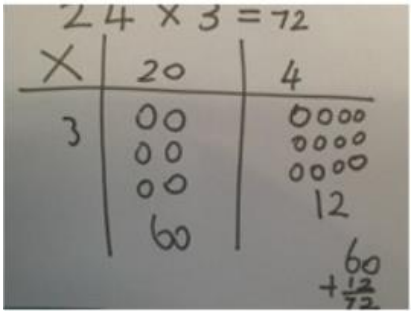
Explore with numbers

Allow children to solve more complicated problems and explore number using what they know, finding multiple solutions to one problem.

E.g. [ ] x [ ] = 240. Find all of the possibilities.

Concrete → Pictorial → Abstract example:

24 x 3 = 72

Concrete:	Pictorial:	Abstract:
(3 rows of 24) 		24 x 3 = 72 24 + 24 + 24 = 72 Explore related facts: 24 x 3 = 72 3 x 24 = 72 72 ÷ 3 = 24 72 ÷ 24 = 3

Key vocabulary:

(Y1) groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals
(Y2) times, multiplication, inverse, operation, equal groups, times as big as, column, row
(Y3) partition, grid method, multiple, product, tens, ones, 2-digit number, value
(Y4) sets of, commutative, column method

Key skills:

- Count in multiples of 6, 7, 9, 25 and 1000.
- Recall multiplication facts for all multiplication tables up to 12 x 12.
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutative law and other strategies mentally 2 x 6 x 5 = 10 x 6 , 39 x 7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Solve problems involving scaling e.g. if 2 children have 3 cakes, how many will 6 children have?

Division:

Mental strategies

Children should use known facts to help them to calculate TO ÷ O calculations.
They may use their knowledge of multiples of 6, 7, 9, 25 and 1000 to answer division problems such as 63 ÷ 9 → 9, 18, 27, 36, 45, 54, 63 → 7 lots of 9.
They may partition the number into manageable chunks e.g. 45 ÷ 3 → 30 ÷ 3 = **10**, 15 ÷ 3 = **5** so 45 ÷ 3 = **15**

Division facts

Once children have learned their times table facts up to 12x12, they should be able to use these facts to answer corresponding division facts through recall. They can also use Times Table Rock Stars to practise.
E.g. I know that 7 x 6 = 42, so I should also recognise the answer to 42 ÷ 7 and 42 ÷ 6.

Using known facts (÷/x 10, ÷/x 100)

Children can also use these known times table facts to answer larger and smaller scaled questions accordingly.
E.g. If I know that 45 ÷ 9 = 5, I also know that 450 ÷ 9 = 50, that 45 ÷ 90 = 0.5 and more.

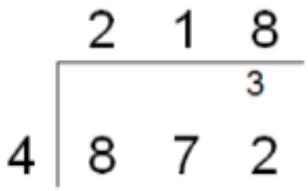
Sharing/grouping

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

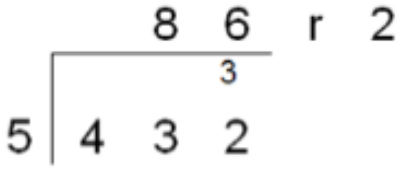
1. Dividend (number to be divided) just over 10x the divisor, e.g. 84÷7
2. Dividend just over 10x the divisor when the divisor is a teen number e.g. 173÷15
3. Dividend over 100x the divisor, e.g. 840÷7
4. Dividend over 20x the divisor, e.g. 168÷7

All the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context (i.e. rounded up or down to relate to the answer to the problem)



Short division

Children should be taught the written division method. The child would need to be confident with their times table knowledge and understand how the dividend (96) is shared by the divisor (3) to give the answer (quotient).
This also needs to include remainders.



Missing number problems

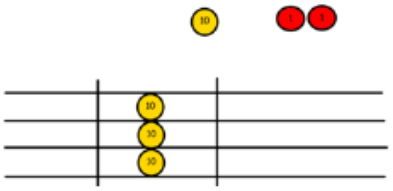
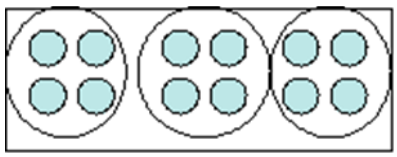
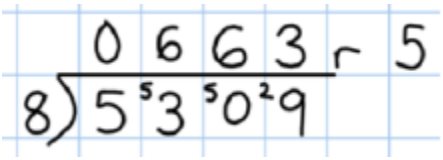
Children should be able to progress to find missing numbers in a variety of different division calculations.

720 ÷ 8 = □	□ = 720 ÷ 8	720 ÷ □ = 8	8 = 720 ÷ □
□ ÷ 8 = 90	8 = □ ÷ 90	□ ÷ 90 = 8	90 = □ ÷ 8

Explore with numbers

Children should be given the chance to explore with numbers
E.g. True or false? When you divide a 3-digit number by a 1-digit number, you always get a 2-digit answer. Explain your reasoning.

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. 	 **Grouping**	 **Short division**

Key vocabulary:

(Y1) share, share equally, __ each, groups, groups of, lots of, array, calculation, amount
(Y2) divide, divided by, divided into, division, grouping, number line, left, left over
(Y3) inverse, short division, carry, multiple
(Y4) quotient, dividend, divisor, remainder

Key skills:

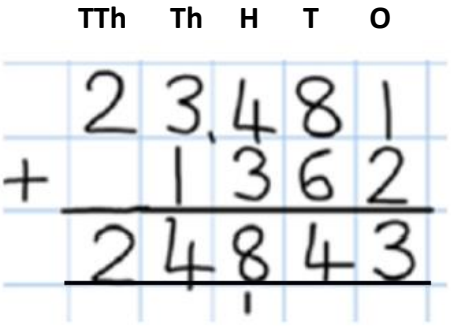
- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Year Five

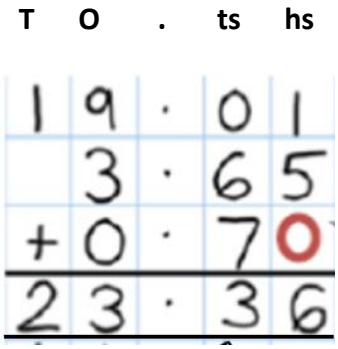
Addition:

Mental strategies
Children should still be using mental strategies built up over the years, such as doubles, near doubles (6 + 7), counting on in ones, tens, hundreds, thousands and tens of thousands from any given number and number bonds to 10, 20, 50, 100 and 1000.

Formal column method (add numbers with more than 4 digits)
Following on from Year 4, children should be using column method for addition regularly. They should be well versed in the language that goes with this, such as '6 hundred + 8 hundreds = 14 hundreds, which is one thousand (ten hundreds) and 4 hundreds.' Children should progress to adding up numbers that have more than 4 digits in them, using the same method taught previously and still using their place value headings to help align the numbers correctly.



Adding decimals
In Year 5, children should be able to add up numbers that include decimals, this also includes adding whole numbers and decimals, or numbers with tenths to numbers with tenths and hundredths. Children may want to use decimal place value counters to help them with this, alongside a place value mat with decimal headings. Numbers must be aligned carefully, using 0 place value holders when there is a blank space, such as 0 hundredths in 0.7
E.g. 19.01 + 3.65 + 0.7 is modelled here →

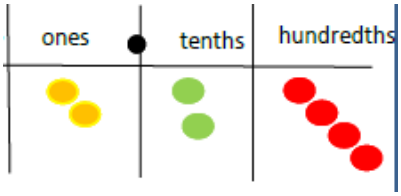
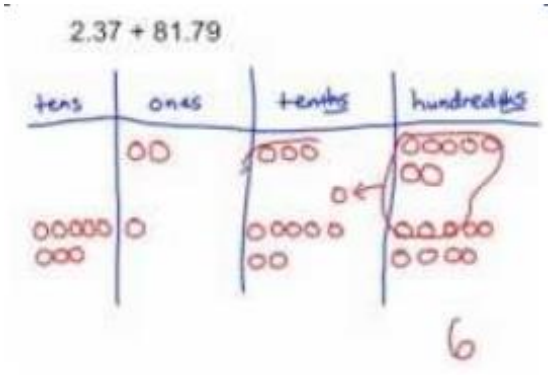


Choosing the best method
It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children.
E.g. 899 + 342 → Much better to take 1 from the 342 to make 900 + 341 = 1241 than to use column method.

Missing number calculations
Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 36,283 = 14,729 + 21,554 21,554 + 14,729 = 29,509 + 6,774
All possibilities of missing numbers shown to children to they understand how these numbers connect.
E.g. 36,283 – 14,729 = [] [] = 36,283 - 14,729 14,729 + [] = 36,283 36,283 = [] + 14,729

Explore with numbers
Give children chance to solve problems or find lots of solutions to one problem.
E.g. 1 Gucci bag and 2 gold chains add up to £5,290.
Both the bag and the gold chains cost more than £1,250 each.
How much could the Gucci bag and the gold chains cost? Find all possibilities.

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
		T O . ts hs 2 6 . 3 4 + 4 5 . 1 7 7 1 . 5 1 1 1

Key vocabulary:

(Y1) add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation
(Y2) sum, tens, ones, partition, units, addition, column, exchange, number line
(Y3) tens boundary, hundreds boundary, carry over, increase, vertical, expanded, inverse
(Y4) compact, thousands, tenths, hundredths, digits, value
(Y5) decimal point, decimal place, thousandths

Key skills:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies e.g. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of column addition.

Subtraction:

Mental strategies

Children should still be using mental strategies built up over the years, such as halving, counting back in ones, tens, hundreds, thousands and tens of thousands from any given number and number bonds to 10, 20, 50, 100 and 1000.

Formal column method (subtract numbers with more than 4 digits)

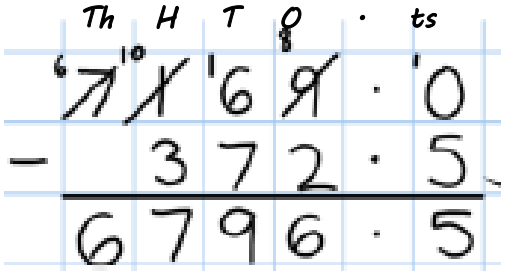
Following on from Year 4, children should be using column method for subtraction regularly. They should be well versed in the language that goes with this, such as '6 hundred - 8 hundreds…. This doesn't work, we need more hundreds. So I'll exchange one of the thousands for 10 hundreds, making 16 hundreds. 16 hundreds – 8 hundreds = 8 hundreds.'

Children should progress to subtracting numbers that have more than 4 digits in them, using the same method taught previously and still using their place value headings to help align the numbers correctly.

Subtracting decimals

In Year 5, children should be able to subtract numbers that include decimals, this also includes subtracting decimals from whole numbers or numbers with tenths/numbers with tenths and hundredths. Children may want to use decimal place value counters to help them with this, alongside a place value mat with decimal headings. Numbers must be aligned carefully, using 0 place value holders when there is a blank space, such as 0 tenths in 7,169.

E.g. 7,169 – 372.5 is modelled here →



Choosing the best method

It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children.

E.g. 4,003 – 2,279 → Much better to take 4 off each number, to leave a better calculation to work with for column method. → 3,999 – 2,275 = 1,724.

Missing number calculations

Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 24,827 = 29,873 – 4,046 12,847 - 1,744 = 15,246 – 4,143

All possibilities of missing numbers shown to children to they understand how these numbers connect.

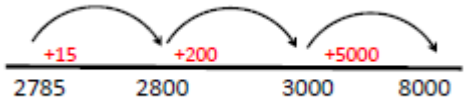
E.g. 11,583 – 5,486 = [] [] = 11,583 – 5,486 5,486 + [] = 11,583 11,583 = [] + 5,486

Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.

E.g. The difference in cost for a book and a pen is £7.45. The book and pen cost £11.40 altogether. How much do the book and the pen cost?

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
	Count on from the lower value 	$\begin{array}{r} {}^{6}\not{7}\,{}^{1}2\,{}^{4}\not{5}\cdot{}^{1}75 \\ -\;233\cdot82 \\ \hline 491.93 \end{array}$

Key vocabulary:

(Y1) equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation
(Y2) difference, count on, strategy, partition, regroup, exchange, tens, ones
(Y3) decrease, inverse, digit, value, method, hundreds
(Y4) thousands, tenths, hundredths, decimal
(Y5) decimal point, tens of thousands

Key skills:

- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Multiplication:

Mental strategies

Children should rely on known facts and counting to help them with their multiplication.
Children should be able to become familiar with prime numbers, square numbers and cube numbers. By the end of Year 5, they should know the first few of each of these by heart.

Times table recall

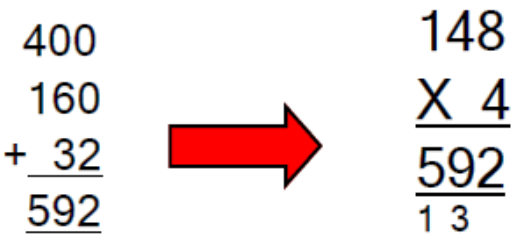
Children are expected to know all of their times table and division facts up to 12 x 12 by the time they reach the end of Year 4. In order to make sure all children have achieved this and are able to retain these facts and apply them to other multiplication problems, they need to know and remember the facts, rather than spend the time counting up or drawing a diagram to help them. However, children will need to spend time looking at the facts alongside diagrams, repeating out loud, using songs and so on to learn them efficiently. Knowing these times table facts should also include divisions. Children can get practise with these tables by using Times Table Rock Stars.

x by 10, 100 and 1000

Children should be able to understand how numbers move up through the place value system when multiplying by 10, 100 or 1000. This includes decimal numbers.
E.g. 34.6 x 100 → 3 tens jumps up 2 PV places to thousands. The rest of the numbers follow this same pattern, keeping one next to another. So 34.6 x 100 = 3,460. 0 is used as a place holder in the ones column. This can be practised using place value charts.

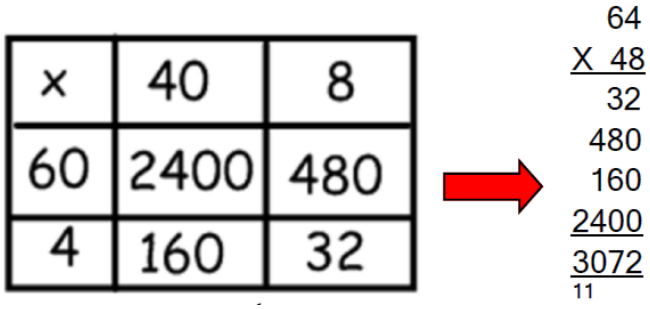
Short multiplication

Following on from Year 4, children in Year 5 should review partitioning method alongside column method to build up an understanding of how to two relate. Short multiplication should be done using the expanded method first, moving on to the full column method when children feel more confident.
Carried numbers should go below the answer box.



Long multiplication

Once children feel confident with short multiplication, they can progress on to using long multiplication for 2d x 4d calculations. This relies on their place value knowledge, their times table knowledge and being able to calculate and keep a lot of information in their heads before calculating. Using grid method from Y3 onwards may help to break it up into stages before putting it all together in long multiplication.



Approximating and estimating to minimise error

When looking at a calculation, children should be able to estimate what the answer should be close to before calculating. This involves making an approximation e.g. 287 x 48 → 300 x 50 = 15,000.
They should record this approximation to check their final answer against. This will help children to see if they are out by powers of 10. For instance, an answer of 1,456 would definitely have an errors in it.

Missing number problems

Using understanding of the inverse and practical resources to solve missing number problems.
Children need to practise this alongside division as well to know how to find the inverse.

25 x 16 = □ □ = 16 x 25 16 x □ = 400 400 = □ x 25

Explore with numbers

Allow children to solve more complicated problems and explore number using what they know, finding multiple solutions to one problem.

E.g. 0. [ ] x 9 → No matter what single digit from 1-9 is put in that box, the answer will always be less than 9. True or false? How do you know?

Concrete → Pictorial → Abstract example:

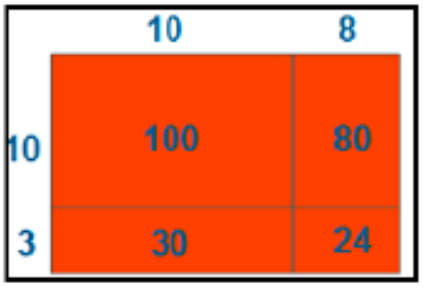
18 x 13 = 234

Concrete:

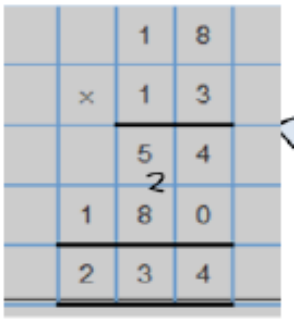
	10	8
10	O	O O O O O O O O
3	O O O	O O O O O

O = 100 O = 10 O = 1

Pictorial:



Abstract:



Key vocabulary:

(Y1) groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals
(Y2) times, multiplication, inverse, operation, equal groups, times as big as, column, row
(Y3) partition, grid method, multiple, product, tens, ones, 2-digit number, value
(Y4) sets of, commutative, column method
(Y5) prime, square, cube, root, integer

Key skills:

- Identify multiples and factors, using knowledge of multiplication tables to 12x12.
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Division:

Mental strategies

Children should use known facts to help them to calculate TO ÷ O calculations.
They may use their knowledge of multiples to answer division problems such as
84 ÷ 12 → 12, 24, 36, 48, 60, 72, 84 → 7 lots of 12.
They may partition the number into manageable chunks e.g. 147 ÷ 7 → 140 ÷ 7 = **20**, 7 ÷ 7 = **1** so
147 ÷ 7 = **21**.

Division facts

Once children have learned their times table facts up to 12x12, they should be able to use these facts to answer corresponding division facts through recall. They can also use Times Table Rock Stars to practise.
E.g. I know that 8 x 6 = 48, so I should also recognise the answer to 48 ÷ 8 and 48 ÷ 6.

÷ by 10, 100 and 1000

Children should be able to understand how numbers move down through the place value system when dividing by 10, 100 or 1000. This includes decimal numbers.
E.g. 762 ÷ 100 → 7 hundreds jumps down 2 PV places to 7 ones. The rest of the numbers follow this same pattern, keeping one next to another after the decimal point. So 762 ÷ 100 = 7.62.
This can be practised using place value charts.

Short division including remainders

184 ÷ 8 = 23 →

```
   2 3
8 | 1 8 ²4
```

Continue to use the language of place value to ensure understanding.

Introduce remainders e.g. 432 ÷ 5 = 86 r 2

```
   8 6 r 2
5 | 4 3 ³2
```

Children should be taught the written division method. The child would need to be confident with their times table knowledge and understand how the dividend (432) is shared by the divisor (5) to give the quotient (86) with the remainder (2).

Remainders as fractions or decimals

Children should be able to begin to express remainders as fractions, using the remainder for the numerator and the divisor as the numerator. There are 2 parts out of the 5 left.
So 432 ÷ 5 = 86 and 2/5.
This can also be converted into a decimal once the children are confident: 432 ÷ 5 = 86.4

Missing number problems

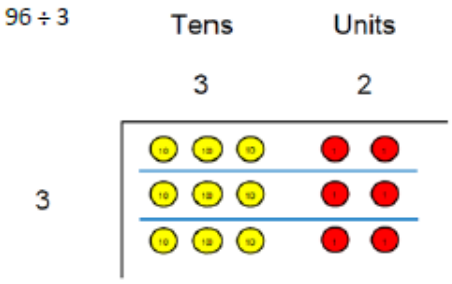
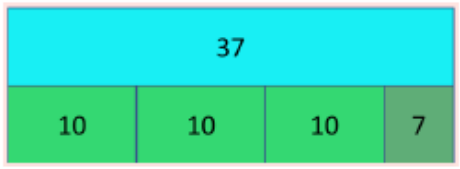
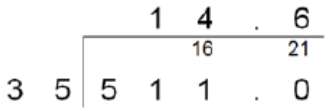
Children should be able to progress to find missing numbers in a variety of different division calculations.

648 ÷ 8 = □	□ = 648 ÷ 8	648 ÷ □ = 8	8 = 648 ÷ □
□ ÷ 8 = 81	8 = □ ÷ 81	□ ÷ 81 = 8	81 = □ ÷ 8

Explore with numbers

Children should be given the chance to explore with numbers
E.g. When you divide a number by 0.1, it get 10 times smaller. True or false? Explain your reasoning with examples.

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
		

Key vocabulary:

(Y1) share, share equally, __ each, groups, groups of, lots of, array, calculation, amount
(Y2) divide, divided by, divided into, division, grouping, number line, left, left over
(Y3) inverse, short division, carry, multiple
(Y4) quotient, dividend, divisor, remainder
(Y5) prime factors, composite numbers

Key skills:

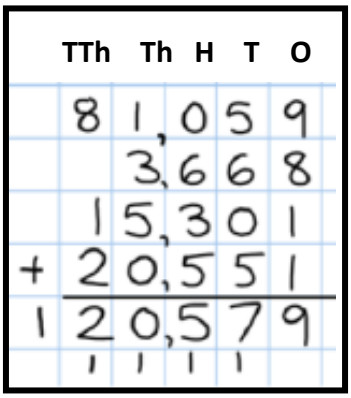
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Year 4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding:
  e.g. $98 \div 4 = 24$ r $2 = 24\frac{1}{2} = 24.5$.
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple ratios.

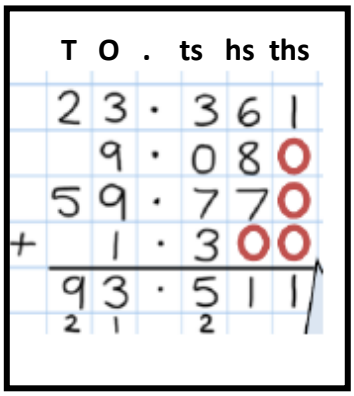
Year Six

Addition:

Mental strategies
Children should still be using mental strategies mentioned in previous year groups.
They should be able to pick and choose which strategies are best depending on the question presented to them.

Adding several numbers with increasingly complexity
By Year 6, children should be using a formal column method to add up several numbers. These numbers may be large numbers, a mix of larger and smaller numbers, numbers with a range of decimal places or a combination of whole numbers and numbers with decimals. The same layout is expected, with place value heading being used to align the numbers and to cement their understanding of place value.





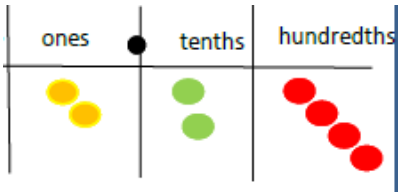
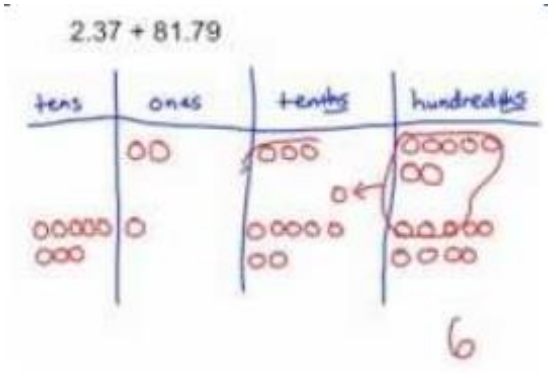
Place value zeros need to be added in for decimal numbers with varying number of decimal places.
Lining up the decimal point is a good technique for adding up these decimal numbers.

Choosing the best method
It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children. Being able to choose the best method will help children to calculate quickly and prepare them well for any exam time constraints.
E.g. 1.98 + 2.09 + 3.95 → Use the 9 hundredths (0.09) and split into 2 lots of 0.02 (2 hundredths) and 0.05 (5 hundredths). Use 0.02 and 0.05 to make 1.98 into 2 and 3.95 into 4.
→ 2 + 2 + 4 + 0.02 = 8.02. This mental strategy relies on a core understanding of number, rounding, partitioning and placement. However, when understood, is far more efficient than using column method for questions like these.

Missing number calculations
Calculations should be written either side of the equals sign so that the sign is not just interpreted as 'the answer'. E.g. 40,283 = 16,729 + 23,554 23,554 + 16,729 = 29,509 + 10,774
All possibilities of missing numbers shown to children to they understand how these numbers connect.
E.g. 386,283 – 14,729 = [] [] = 386,283 - 14,729 14,729 + [] = 386,283 386,283 = [] + 14,729

Explore with numbers
Give children chance to solve problems or find lots of solutions to one problem.
E.g. a + b + c = 6.36
b is twice the size of a.
c is the total of a + b.
Calculate the values for a, b and c.

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
		T O . ts hs 2 6 . 3 4 + 5 . 1 0 3 1 . 4 4 1

Key vocabulary:

(Y1) add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, calculation
(Y2) sum, tens, ones, partition, units, addition, column, exchange, number line
(Y3) tens boundary, hundreds boundary, carry over, increase, vertical, expanded, inverse
(Y4) compact, thousands, tenths, hundredths, digits, value
(Y5) decimal point, decimal place, thousandths
(Y6) integer, unknown value

Key skills:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.
- Use negative numbers in context, and calculate intervals across zero.

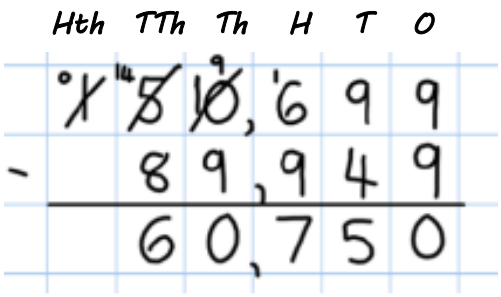
Subtraction:

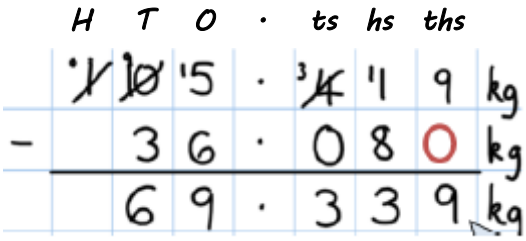
Mental strategies

Children should still be using mental strategies mentioned in previous year groups.
They should be able to pick and choose which strategies are best depending on the question presented to them.

Formal column method (subtract numbers with more than 4 digits)

By Year 6, children should be using a formal column method to subtract several numbers. These numbers may be large numbers, a mix of larger and smaller numbers, numbers with a range of decimal places or a combination of whole numbers and numbers with decimals. The same layout is expected, with place value heading being used to align the numbers and to cement their understanding of place value. Place value zeros need to be added in for decimal numbers with varying number of decimal places. Lining up the decimal point is a good technique for subtracting these decimal numbers. Exchanging should be done with all the correct vocabulary taught in previous years, such as '*we cannot take 8 hundredths from 1 hundredth and leave a positive value, so we need to exchange 1 tenth into 10 hundredths. 4 tenths become 3 tenths and that tenth is used to make 11 hundredths.'*





Choosing the best method

It is important discussing with children which is the best method for the question. Once they have learned column method, they may become reliant on it. Discuss the most efficient methods with the children.
E.g. 65,005 – 12,972 → Much better to take 6 off each number, to leave a better calculation to work with for column method. → 64,999 – 12,966 = 52,033.

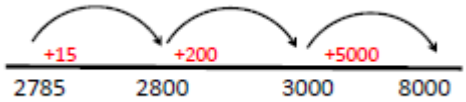
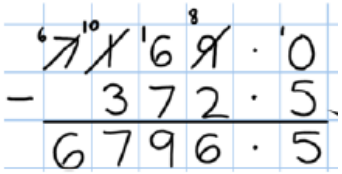
Missing number calculations

Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'. E.g. 204,827 = 209,873 – 4,046 32,847 - 11,744 = 35,246 – 14,143
All possibilities of missing numbers shown to children to they understand how these numbers connect.
E.g. 231,583 – 9,486 = [] [] = 231,583 – 9,486 9,486 + [] = 231,583 231,583 = [] + 9,486

Explore with numbers

Give children chance to solve problems or find lots of solutions to one problem.
E.g. The difference in cost for a back pack and an exercise book is £16.45. The back pack and exercise book cost £20.40 altogether.
How much do the back pack and the exercise book cost?

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
	Count on from the lower value 	

Key vocabulary:

(Y1) equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, count back, how many left, how much less is, calculation
(Y2) difference, count on, strategy, partition, regroup, exchange, tens, ones
(Y3) decrease, inverse, digit, value, method, hundreds
(Y4) thousands, tenths, hundredths, decimal
(Y5) decimal point, tens of thousands
(Y6) integer, unknown value

Key skills:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Multiplication:

Mental strategies

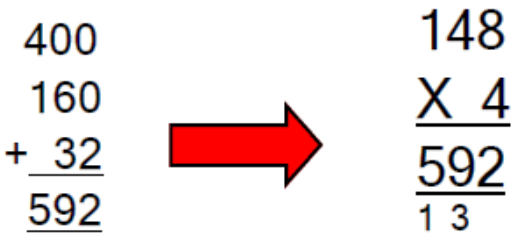
Children should rely on known facts and counting to help them with their multiplication.
Children should be increasingly more familiar with prime numbers, square numbers and cube numbers.
They should be able to apply what they know to a variety of different calculations e.g.
500 x 16 → Partition: 5,000 + 3,000 = 8,000; double 500 to 1000 and half 16 to 8 so 1000 x 8 = 8,000.

Times table recall

Children are expected to know all of their times table and division facts up to 12 x 12 by the time they reach the end of Year 4. In order to make sure all children have achieved this and are able to retain these facts and apply them to other multiplication problems, they need to know and remember the facts, rather than spend the time counting up or drawing a diagram to help them. However, children will need to spend time looking at the facts alongside diagrams, repeating out loud, using songs and so on to learn them efficiently. Knowing these times table facts should also include divisions. Children can get practise with these tables by using Times Table Rock Stars.

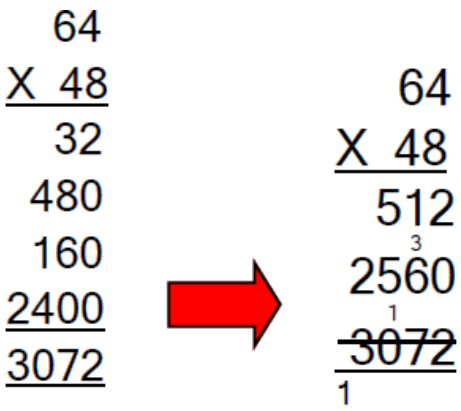
Short multiplication

Following on from Year 5, children in Year 6 should review the short multiplication method with increasingly larger numbers multiplied by a single digit number. Carried numbers should go below the answer box. Some children may need to review Y4/5 method of partitioning and adding up before confidently carrying over as they multiply in the short column method.



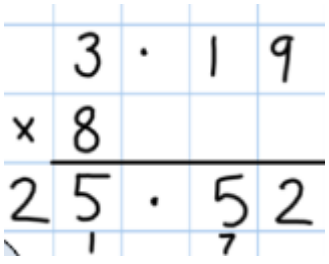
Long multiplication

Once children feel confident with short multiplication, they can progress on to using long multiplication for 2d x 4d calculations. This relies on their place value knowledge, their times table knowledge and being able to calculate and keep a lot of information in their heads before calculating. Using an expanded version of long multiplication is best to start with, allowing children to build up their confidence before moving on to the final compacted method.



Multiplying with decimal numbers

In Year 6, children should be able to multiply a whole number by a decimal number. By ensuring the ones are aligned and place value is followed, they can use their existing knowledge of short multiplication method and apply this to calculations like the one shown here →
It is best to get children to estimate their answer first to allow them to easily identify any errors made with place value.
E.g. 3.19 x 8 is approximately 3 x 8 = 24 so the answer should be close to 24.



Approximating and estimating to minimise error

Just like in Year 5, children should be able to estimate what the answer should be close to before calculating. This involves making an approximation e.g. 287 x 48 → 300 x 50 = 15,000.

They should record this approximation to check their final answer against. This will help children to see if they are out by powers of 10. For instance, an answer of 1,456 would definitely have an errors in it.

Missing number problems

Using understanding of the inverse and practical resources to solve missing number problems.

Children need to practise this alongside division as well to know how to find the inverse.

75 x 16 = □ □ = 16 x 75 16 x □ = 1,200 1,200 = □ x 25

Explore with numbers

Allow children to solve more complicated problems and explore number using what they know, finding multiple solutions to one problem.

E.g. 1[ ] x [ ]8 = 532.

What are the missing numbers? What is the most efficient method of finding the answer?

Concrete → Pictorial → Abstract example:

Concrete:

	30	7
10	O O O	O O O O O O O
9	O O O O O O O O O	O O O O O O

O = 100 O = 10 O = 1

Pictorial:

x	300	20	7
4	1200	80	28

Abstract:

53.2 x 24

```
   1
  53.2
x 24.0
 212.8
1064.0
1276.8
```

Key vocabulary:

(Y1) groups of, lots of, equal groups, array, altogether, multiply, count, calculation, makes, equals
(Y2) times, multiplication, inverse, operation, equal groups, times as big as, column, row
(Y3) partition, grid method, multiple, product, tens, ones, 2-digit number, value
(Y4) sets of, commutative, column method
(Y5) prime, square, cube, root, integer
(Y6) tenths, hundredths, decimal point

Key skills:

- Recall multiplication facts for all times tables up to 12 x 12 (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

Division:

Mental strategies

Children should use known facts to help them to calculate TO ÷ O calculations.
They may use their knowledge of multiples to answer division problems such as
96 ÷ 12 → 12, 24, 36, 48, 60, 72, 84, 96 → 8 lots of 12.
They may partition the number into manageable chunks e.g. 294 ÷ 7 → 280 ÷ 7 = **40**, 14 ÷ 7 = **2** so 294 ÷ 7 = **42**.

Division facts

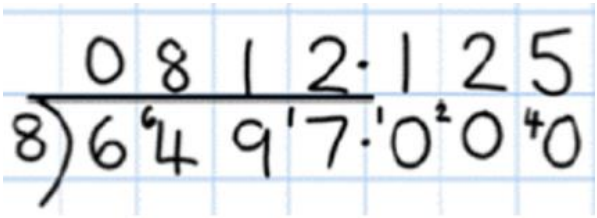
Once children have learned their times table facts up to 12x12, they should be able to use these facts to answer corresponding division facts through recall. They can also use Times Table Rock Stars to practise.
E.g. I know that 11 x 6 = 66, so I should also recognise the answer to 66 ÷ 11 and 66 ÷ 6.

÷ by 10, 100 and 1000

Children should be able to understand how numbers move down through the place value system when dividing by 10, 100 or 1000. This includes decimal numbers.
E.g. 1,483 ÷ 100 → 1 thousands jumps down 2 PV places to 1 ten. The rest of the numbers follow this same pattern, keeping one next to another after the decimal point. So 1,483 ÷ 100 = 14.83.
This can be practised using place value charts.

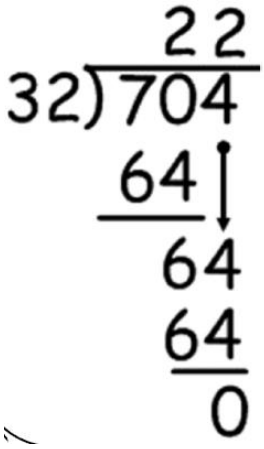
Short division

Children should be taught the written division method.
The child would need to be confident with their times table knowledge and understand how the dividend (6497) is shared by the divisor (8) to give the quotient (812.125 or 812 and 1/8).
They may need to add place value zeros after the dividend to ensure the numbers not divisible can be carried over.



Long division

Year 6 children will start to look at an expanded method of long division. This allows them to work with smaller amounts at one time and understand what they are doing and why, rather than just learning a method. Children are taught to work across the place value columns, asking themselves each time, 'What is the biggest multiple of the divisor I can take away?'. Write the size of the 'chunk' of the divisor on the answer line, then subtract the multiple. Once the subtraction is completed, bring down the next digit from the answer and repeat the process. As with short division, answers can be given as a whole number with a remainder, as a mixed number or with a decimal depending on the context.



Missing number problems

Children should be able to progress to find missing numbers in a variety of different division calculations.

729 ÷ 9 = □ □ = 729 ÷ 9 729 ÷ □ = 9 9 = 729 ÷ □

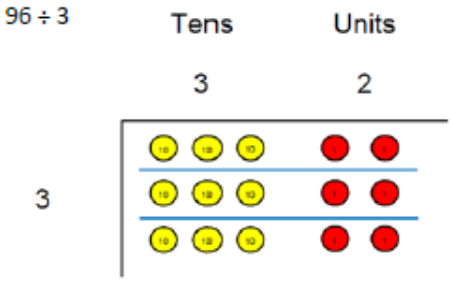
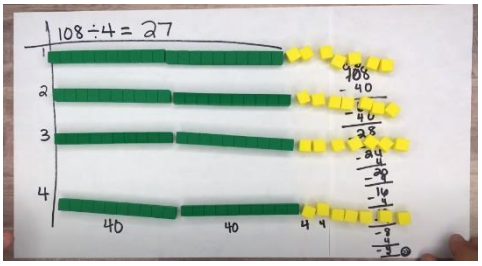
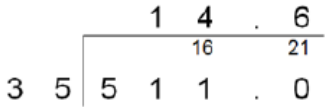
□ ÷ 9 = 81 9 = □ ÷ 81 □ ÷ 81 = 9 81 = □ ÷ 9

Explore with numbers

Children should be given the chance to explore with numbers

E.g. When you divide a number by 0.01, it get 100 times smaller. True or false? Explain your reasoning with examples.

Concrete → Pictorial → Abstract example:

Concrete:	Pictorial:	Abstract:
		

Key vocabulary:

(Y1) share, share equally, __ each, groups, groups of, lots of, array, calculation, amount
(Y2) divide, divided by, divided into, division, grouping, number line, left, left over
(Y3) inverse, short division, carry, multiple
(Y4) quotient, dividend, divisor, remainder
(Y5) prime factors, composite numbers
(Y6) common factor, divisibility

Key skills:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.